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Collective electronic excitations in a semiconductor superlattice in the Landau and Wannier-Stark ladder regime

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Abstract. Using a mean-field approximation, we have developed a systematic treatment of collective electronic modes in a semiconductor superlattice (SL) in the presence of strong electric and magnetic fields parallel to the SL axis. The spectrum of collective modes with zero wavevector along the SL axis is shown to consist of a principle magnetoplasmon mode and an infinite set of Bernstein-like modes. For non-zero wavevector along the SL axis, in addition to the cyclotron modes, extra collective modes are found at the frequencies $|N\omega_c \pm M\omega_s|$, which we call cyclotron-Stark modes (ω_c and ω_s are respectively the cyclotron and Stark frequencies, N and M are integer numbers). The frequencies of the modes propagating in "oblique" direction with respect to the SL axis show oscillatory behavior as a function of electric field strength. All the modes considered have very weak spatial dispersion and they are not Landau damped. The specific predictions made for the dispersion relations of the collective excitations should be observable in resonant Raman scattering experiments.

PACS. 73.20.Mf Collective excitations (including excitons, polarons, plasmons and other charge-density excitations) – 73.20.Dx Electron states at surfaces and interfaces

1 Introduction

It is well known that an external electric field applied along the growth axis of a semiconductor superlattice (SL) localizes electronic states along the field direction into the now-called Wannier-Stark states, whose extension is inversely proportional to the applied field. This results in a suppression of tunnelling between adjacent quantum wells and therefore corresponds to a splitting of each allowed energy band ("miniband") of a SL into a Wannier-Stark ladder, a set of equally spaced subbands whose separation depends linearly on both the electric-field strength and the period of the SL. Because of the large value of the latter ($\simeq 10$ nm) in a real SL, the Wannier-Stark levels are easily resolved − even in relatively low electric fields ($\simeq$ 10–20 kV/cm). Their existence has been demonstrated by means of transport and optical experiments, involving photocurrent and photoluminescence excitation spectroscopy, as well as electroreflectance (for a review see [1–4]).

The ladder-like energy spectrum of the SL is still more specific in the presence of a magnetic field applied perpendicular to the layers, that is parallel to the electric field. Due to the localizing effect of the magnetic field in the plane of the SL parallel to the interface, the electronic states of this system are localized in all three dimensions, and hence have purely discrete energies, like those occurring in an atomic system or in a quantum-dot structure. This leads to a series of interesting physical phenomena whose investigation is becoming a very active area of research [5–22].

These studies were mainly concentrated on single-particle aspects of the problem, with little attention paid to the consequences of many-body effects which arise due to electron-electron interactions (the only exception we are aware of is the paper of Barticevic *et al.* [23]). Among the most interesting of these is the existence of the collective excitations of an electron gas, in particular the plasma oscillations and the dielectric screening associated with them. While over the last two decades the collective excitations in a SL have been extensively studied both theoretically and experimentally by many authors (see reviews [24–26] and the recent papers [27,28] for references), there have been no reports on the problem of collective excitations when an electric field and a magnetic field are applied to the system simultaneously. Meanwhile the two field configurations for a SL are very interesting since under these conditions new branches of the collective excitation spectrum can be expected to arise. In the present

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paper we focus our attention on the theoretical investigation of collective electronic modes in a SL in the presence of electric and magnetic fields along the SL axis.

The rest of the paper is organized as follows. In Section 2 we describe our model and develop the constitutive equations for the theory of the collective excitations in a SL subject to parallel electric and magnetic fields. In Section 3 we apply our theory to the case of low densities of an electron gas in a SL and derive the dispersion relations for collective excitations of the system. In this section, numerically calculated dispersion curves are also presented and discussed in detail. Finally, we give a summary of our main conclusions in Section 4.

2 Description of the model and derivation of the RPA dispersion equation

The model adopted in this paper to describe the SL structure is standard and has previously been used by many investigators to study collective electronic modes of a SL since the pioneering works of Bloss and Brody [29], Das Sarma and Quinn [30] and Tselis and Quinn [31]. It presents a periodic array of two-dimensional electron gas layers separated from each other by a distance d which is the period of the SL, and are embedded in a neutralizing background with dielectric constant κ_0. The so-called type-I superlattices (such as GaAs/Al_xGa_{1-x}As) consisting of alternating layers of two semiconductors with band-gap discontinuity at the interface of the two materials and with sufficiently close dielectric constants can be considered as a prototype of this model system. It is assumed that the system is infinitely extended in the z direction parallel to the SL axis and subject to an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ along the same direction.

It is convenient to describe the magnetic field $\mathbf{B}$ by the vector potential $\mathbf{A}$ using the relation $\mathbf{B} = \nabla \times \mathbf{A}$ in the Landau gauge

$$A_x = A_z = 0, \quad A_y = Bx. \tag{1}$$

In this case the Hamiltonian of an electron is given by

$$H_e = \frac{p_x^2}{2m_\perp} + \frac{(p_y + m_\perp \omega_c x)^2}{2m_\perp} + \frac{p_z^2}{2m_\parallel} + U(z) + eEz, \tag{2}$$

where $\mathbf{p}(p_x, p_y, p_z)$ is the momentum operator of a conduction electron, $\omega_c = eB/m_\perp c$ is the frequency of the cyclotron rotation in the x-y plane and $U(z) = U(z + d)$ is the SL periodic potential which confines electrons to the planes $z = md$ (m is the integer labelling the mth layer). It is assumed that the period of the SL is considerably larger than the lattice constants of the host semiconductors which compose the SL. This enables the envelope-function approximation [32–35] to be used for describing the electronic structure of the SL, where the periodic potential of the host materials is taken into account through the effective electron conduction-band-edge masses $m_\parallel$ and $m_\perp$ in the direction along and perpendicular to the SL axis, respectively.

The eigenfunctions $\psi_{k\lambda}(\mathbf{r})$ and eigenvalues ε_λ of H_e can be cast in the form

$$\psi_{k\lambda}(\mathbf{r}) = \frac{1}{\sqrt{L_y}} \exp(\mathrm{i}ky)\phi_{lk}(x)\chi_n(z), \tag{3}$$

$$\varepsilon_\lambda \equiv \varepsilon_{ln} = \varepsilon_l + \varepsilon_n = \left(l + \frac{1}{2}\right)\hbar\omega_c + n\hbar\omega_s. \tag{4}$$

Note that in writing equation (4) we have chosen the zero level of energy at the energy of the lowest (ground) miniband in the absence of both a DC magnetic field and a DC electric field. In equations (3, 4) the following notations are used: L_y is the size of the system in the y direction, k is the y-component of the electron wavevector, λ is the composite index (l, n) consisting of a Landau level quantum number $l(= 0, 1, 2, \dots)$ and the Wannier-Stark level index $n(= 0, \pm1, \pm2, \dots)$, $\omega_s = eEd/\hbar$ is the Wannier-Stark frequency, $\chi_n(z)$ is the envelope function, and $\phi_{lk}(x)$ is the normalized harmonic-oscillator wavefunction centered at $x_0 = -a_B^2 k$, having the form

$$\phi_{lk}(x) \equiv \phi_l\left(\frac{x - x_0}{a_B}\right) = \left(\frac{1}{\sqrt{\pi}2^l l! a_B}\right)^{1/2} \times \exp\left\{-\frac{1}{2}\left(\frac{x - x_0}{a_B}\right)^2\right\} H_l\left(\frac{x - x_0}{a_B}\right). \tag{5}$$

Here $a_B = (\hbar/m_\perp\omega_c)^{1/2}$ is the radius of the cyclotron orbit in the x-y plane and $H_l(x)$ is the Hermite polynomial of the lth order.

The envelope function $\chi_n(z)$ describing the quantized electron motion along the SL axis is given by the well-known expression [1,4]

$$\chi_n(z) = \sum_{m=-\infty}^{\infty} J_{m-n}(\Delta/2\hbar\omega_s)\varphi(z - md), \tag{6}$$

where $\varphi(z - md)$ is the quantum-well eigenstate centered at md, $J_n(\Delta/2\hbar\omega_s)$ is the Bessel function of the first kind and of integer n, and Δ is the width of the lowest miniband of a SL. Since the width of the electron density profile in any one quantum well is assumed to be much less than the SL period d, the function $|\varphi(z - md)|^2$ may be presented as the Dirac delta function located at $z_m = md$ [29–31]

$$|\varphi(z - md)|^2 = \delta(z - md). \tag{7}$$

This approximation seems to be reasonable for fairly low electron densities which will concern us in this paper. Following the approach developed in references [36,37], it is not hard to show that changing $|\varphi(z-md)|^2$ to more complicated localized functions does not change the conclusions of this paper in any qualitative fashion.

It should be pointed out that Landau and Wannier-Stark state levels cannot be resolved when their thermal and collision broadening is grater than their energy separation. Therefore, we assume that the following inequalities are valid

$$\hbar\omega_c(\omega_s) \gg k_B T, \quad \omega_c(\omega_s)\tau \gg 1, \tag{8}$$

$$H = H_0 + H_{e-e} = \sum_{k\lambda} \varepsilon_\lambda a^\dagger_{\lambda k} a_{\lambda k} + \frac{1}{2} \sum_{k,k',\mathbf{q}_\perp} \sum_{\lambda_{1,2,3,4}} V_{\lambda_1\lambda_2\lambda_3\lambda_4}(k,k',\mathbf{q}_\perp) a^\dagger_{\lambda_1 k+q_y} a^\dagger_{\lambda_2 k'-q_y} a_{\lambda_3 k'} a_{\lambda_4 k}, \quad (13)$$

where k_B is the Boltzmann constant, T is the absolute temperature, and τ is the electron relaxation time. In addition, the restriction to the lowest miniband made above requires the conditions

$$\Delta_g \gg \hbar\omega_s, \quad \Delta_g \gg \hbar\omega_c \quad (9)$$

to be fulfilled (Δ_g is the width of the first minigap in the energy band structure of a SL). Under these conditions we can neglect the magnetic breakdown and Zener tunnelling of electrons from the lowest miniband to higher ones. The possibility of the experimental realization of all the above conditions has been demonstrated in a number of works (see, for example, Ref. [4] where extended references are given).

Let $\mathbf{r}_\parallel$ denote a two-dimensional coordinate vector of an electron in an arbitrary layer of our model SL structure, that is, $\mathbf{r}_\parallel = \mathbf{e}_x x + \mathbf{e}_y y$, where $\mathbf{e}_x$ and $\mathbf{e}_y$ designate the unit vectors directed along the corresponding axes. Electrons in different layers interact only *via* the Coulomb interaction. As mentioned above, the dielectric constants of the two materials forming the SL are assumed to be very close. Therefore, we can neglect any image potential term in the interaction and put the three-dimensional Coulomb potential $V(\mathbf{r}-\mathbf{r}')$ with $\mathbf{r} = (\mathbf{r}_\parallel, z)$ and $\mathbf{r}' = (\mathbf{r}'_\parallel, z')$ in the form

$$V(\mathbf{r}-\mathbf{r}') = \frac{e^2}{\kappa_0 \sqrt{|\mathbf{r}_\parallel - \mathbf{r}'_\parallel|^2 + (z-z')^2}}. \quad (10)$$

Its Fourier transform with respect to $\mathbf{r}_\parallel - \mathbf{r}'_\parallel$ is

$$V(\mathbf{q}_\perp; z-z') = V(\mathbf{q}_\perp) \exp(-q_\perp |z-z'|) \quad (11)$$

with

$$V(\mathbf{q}_\perp) = \frac{2\pi e^2}{\kappa_0 |\mathbf{q}_\perp| L_x L_y}, \quad (12)$$

where $\mathbf{q}_\perp$ is a two-dimensional wavevector in the x-y plane, $L_x L_y$ is the area of each layer of the SL system.

Using the single-particle states (3) as the basis for second quantization, the full Hamiltonian of the system which we are interested in can be expressed as

see equation (13) above,

where $a^\dagger_{\lambda k}$ and $a_{\lambda k}$ are the electron creation and annihilation operators corresponding to the single-electron state $|\lambda k\rangle$ and $V_{\lambda_1\lambda_2\lambda_3\lambda_4}(k,k',\mathbf{q}_\perp)$ denotes the Coulomb interaction vertex

$$V_{\lambda_1\lambda_2\lambda_3\lambda_4}(k,k',\mathbf{q}_\perp) = V(\mathbf{q}_\perp) F_{l_1 l_4}(k+q_y, k; q_x) \times F_{l_2 l_3}(k'-q_y, k'; -q_x) I_{n_1 n_2 n_3 n_4}(q_\perp), \quad (14)$$

in which

$$F_{ll'}(k,k';q_x) = \int \mathrm{d}x\, \phi_{lk}(x) \phi_{l'k'}(x) \exp(\mathrm{i} q_x x), \quad (15)$$

$$I_{n_1 n_2 n_3 n_4}(q_\perp) = \int \mathrm{d}z \int \mathrm{d}z' \chi^*_{n_1}(z) \chi^*_{n_2}(z') \chi_{n_3}(z') \chi_{n_4}(z) \exp(-q_\perp |z-z'|). \quad (16)$$

In the same second quantization representation the electron density operator of the system can be written as

$$\rho(\mathbf{r}) = \sum_{\lambda\lambda'} \sum_{kk'} \psi^*_{\lambda' k'}(\mathbf{r}) \psi_{\lambda k}(\mathbf{r}) a^\dagger_{\lambda' k'} a_{\lambda k}. \quad (17)$$

Its Fourier transform with respect to $\mathbf{r}_\parallel$ is given by

$$\rho(\mathbf{q}_\perp, z) = \sum_k \sum_{\lambda\lambda'} F_{ll'}(k, k+q_y, ; -q_x) \times \chi_n(z) \chi^*_{n'}(z) a^\dagger_{\lambda' k} a_{\lambda k+q_y}. \quad (18)$$

From this equation, we see that the Fourier component of the electron density operator is no longer of the simple form $\sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}+\mathbf{q}}$ as in the three-dimensional free-electron gas where the single-particle states are plane-waves.

To obtain the dispersion relations for the collective modes we consider the linear response of a SL to an external potential. Following Das Sarma and Quinn [30] and Tselis and Quinn [31], we use the normal random-phase approximation (RPA) which includes only direct Coulomb interactions, but ignores exchange contributions as well as all short-range correlations beyond exchange. As it is well known, the RPA is applicable in the weak-coupling regime where $r_s = r_0/a_\mathrm{B} \ll 1$ (r_0 is the mean distance between electrons and a_B is the effective Bohr radius). In this paper we are concerned with a SL where the areal electron density n_s per layer is relatively low ($\sim 10^8 - 10^9$ cm^{-2}), so that $r_s \gg 1$. For such a situation the RPA, which gives the exact results in the high-electron density limit, may not be quite justified and may need to be improved on. Of late, extensive work has been done on the study of the behavior of collective modes (plasmons in this case) beyond RPA (we refer to the Refs. [38,38] on the inclusion of correlation effects on the plasmon spectrum in general and to the Refs. [40–42] in SLs in particular). However, recent experimental and theoretical studies [43, 44] have proved that the plasmon dispersion (up to very large wavevectors) in a low-density (down to $n_s = 5 \times 10^8$ cm^{-2} corresponding to $r_s \sim 25$) two-dimensional electron system in a GaAs quantum-well layer seems to be quantitatively reproduced by RPA, offering much better agreement with experiment [43] compared to its "improved" versions including local-field corrections. We speculate that this is also true for more complicated systems such as a SL, and in our calculation we restrict ourselves to the usual RPA. Of course, in view of the simplifying assumptions made (zero thickness layers, the complete neglect of correlations), all the further predictions regarding the collective mode spectrum for finite wavevectors have to be treated

with some caution. It should be stressed, however, that it is not our objective at this stage to strive towards an agreement with possible experiments.

To begin with, we consider the electron density-fluctuation operator $\delta\rho(\omega, \mathbf{q}_\perp; z)$ which determines the density response of the system to an external harmonic perturbation of frequency ω:

$$\delta\rho(\omega, \mathbf{q}_\perp; z) = \sum_k \sum_{\lambda\lambda'} F_{ll'}(k, k+q_y, ; -q_x) \times \chi_n(z)\chi^*_{n'}(z) \left(a^\dagger_{\lambda' k} a_{\lambda k+q_y}\right)_1 . \quad (19)$$

Here the subscript 1 at the operator $a^\dagger_{\lambda' k} a_{\lambda k+q_y}$ refers to the perturbed part of the operator.

Next, following the self-consistent field prescription, we consider the equation of motion for the operator $a^\dagger_{\lambda' k} a_{\lambda k+q_y}$:

$$-\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t}(a^\dagger_{\lambda' k} a_{\lambda k+q_y}) = \left[H, a^\dagger_{\lambda' k} a_{\lambda k+q_y}\right] = (\varepsilon_{\lambda'} - \varepsilon_\lambda) a^\dagger_{\lambda' k} a_{\lambda k+q_y} + \left[H_{e-e}, a^\dagger_{\lambda' k} a_{\lambda k+q_y}\right]. \quad (20)$$

Applying the standard RPA to the last term in the above equation yields

$$-\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t} \left\langle a^\dagger_{\lambda' k} a_{\lambda k+q_y} \right\rangle_1 = (\varepsilon_{\lambda'} - \varepsilon_\lambda) \left\langle a^\dagger_{\lambda' k} a_{\lambda k+q_y} \right\rangle_1 + \left(f^{(0)}_\lambda - f^{(0)}_{\lambda'}\right) \sum_{k' q_x} \sum_{\lambda_{1,2}} V_{\lambda'\lambda_1\lambda_2\lambda}(k, k'+q_y, \mathbf{q}_\perp) \times \left\langle a^\dagger_{\lambda_1 k'} a_{\lambda_2 k'+q_y} \right\rangle_1 . \quad (21)$$

With a linear response to an external field with frequency ω

$$-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t} \left\langle a^\dagger_{\lambda' k} a_{\lambda k+q_y} \right\rangle_1 = -\omega \left\langle a^\dagger_{\lambda' k} a_{\lambda k+q_y} \right\rangle_1 . \quad (22)$$

Consequently, equation (21) takes the form

$$\langle a^\dagger_{\lambda' k} a_{\lambda k+q_y} \rangle_1 - P_{\lambda\lambda'}(\omega) \sum_{k' q_x} \sum_{\lambda_{1,2}} V_{\lambda'\lambda_1\lambda_2\lambda}(k, k'+q_y, \mathbf{q}_\perp) \langle a^\dagger_{\lambda_1 k'} a_{\lambda_2 k'+q_y} \rangle_1 = 0, \quad (23)$$

where $P_{\lambda\lambda'}(\omega)$ is defined as

$$P_{\lambda\lambda'}(\omega) \equiv P_{ln,l'n'}(\omega) = \frac{f^{(0)}(\varepsilon_{ln}) - f^{(0)}(\varepsilon_{l'n'})}{\varepsilon_{ln} - \varepsilon_{l'n'} + \hbar\omega}. \quad (24)$$

Using equations (5) and (15), it is easy to get the relation

$$\sum_k F_{ll'}(k, k+q_y; -q_x) F_{l'l}(k+q_y, k; q'_x) = \frac{L_x L_y}{2\pi a_B^2} \delta(q_{x'}, q_x) |F_{ll'}(q_\perp a_B)|^2, \quad (25)$$

where $\delta(q_{x'}, q_x)$ is the Kronecker delta, $q_\perp^2 = q_x^2 + q_y^2$, and $|F_{ll'}(q_\perp a_B)|^2$ can be expressed in terms of the associated Laguerre polynomials $L_{l_2}^{l_1-l_2}(x)$,

$$|F_{ll'}(q_\perp a_B)|^2 = \frac{l_2!}{l_1!} \left(\frac{q_\perp a_B}{2}\right)^{l_1-l_2} \exp\left(-\frac{q_\perp a_B}{2}\right) \times \left[L_{l_2}^{l_1-l_2}\left(\frac{q_\perp a_B}{2}\right)\right]^2, \quad (26)$$

with $l_1 = \max(l, l')$ and $l_2 = \min(l, l')$. Taking into account (14–16) and (19), we then get from equation (23) that $\langle \delta\rho(\omega, \mathbf{q}_\perp; z) \rangle$ satisfies the integral equation

$$\langle \delta\rho(\omega, \mathbf{q}_\perp; z) \rangle - \frac{e^2}{\kappa_0 a_B^2 q_\perp} \sum_{\lambda\lambda'} P_{\lambda\lambda'}(\omega) |F_{ll'}(q_\perp a_B)|^2 \times \chi^*_n(z)\chi^*_{n'}(z) \int \mathrm{d}z' \int \mathrm{d}z'' \exp(-q_\perp |z'' - z'|) \times \chi^*_n(z'')\chi^*_{n'}(z'') \langle \delta\rho(\omega, \mathbf{q}_\perp; z') \rangle = 0. \quad (27)$$

In view of the translational symmetry of the SL in the z direction, the following *Ansatz* can be used as a solution for equation (27):

$$\langle \delta\rho(\omega, \mathbf{q}_\perp; z) \rangle = \langle \delta\rho(\omega, \mathbf{q}_\perp) \rangle \exp(\mathrm{i}q_z z) \quad (28)$$

with z being equal to the possible discrete values $z_m = md$ allowed in the SL structure and with the "wave number" q_z continuously varying within the first Brillouin minizone of the SL, *i.e.*, $-\pi/d \le q_z \le \pi/d$. The physical meaning of q_z is quite clear: this quantity determines the relative phase of the electron-density oscillations in the adjacent layers of the SL.

Substituting (28) into equation (27), we then find

$$1 - \frac{e^2}{\kappa_0 a_B^2 q_\perp} \sum_{\lambda\lambda'} P_{\lambda\lambda'}(\omega) |F_{ll'}(q_\perp a_B)|^2 \times \exp(-\mathrm{i}q_z z)\chi^*_n(z)\chi^*_{n'}(z) \int \mathrm{d}z' \exp(\mathrm{i}q_z z') \times \int \mathrm{d}z'' \exp(-q_\perp |z'' - z'|)\chi^*_n(z'')\chi^*_{n'}(z'') = 0. \quad (29)$$

Next, we integrate (29) over z in the limits from 0 to $L_z = Nd$, where N is the number of the SL layers. Using (6) and taking into account that wave functions of electrons in adjacent layers do not overlap (the coupling between the layers is realized only through a mean-field), we get

$$1 - \frac{e^2}{\kappa_0 q_\perp a_B^2} S(q_\perp, q_z) \sum_{\lambda,\lambda'} P_{\lambda\lambda'} |F_{ll'}(q_\perp a_B)|^2 \times \sum_m J_{m-n}\left(\frac{\Delta}{\hbar\omega_s}\right) J_{m-n'}\left(\frac{\Delta}{\hbar\omega_s}\right) \mathrm{e}^{\mathrm{i}mq_z d} \times \sum_{m'} J_{m'-n}\left(\frac{\Delta}{\hbar\omega_s}\right) J_{m'-n'}\left(\frac{\Delta}{\hbar\omega_s}\right) \mathrm{e}^{-\mathrm{i}m'q_z d} = 0, \quad (30)$$

where the function $S(q_\perp, q_z)$ is defined as

$$S(q_\perp, q_z) = \sum_{n'} \exp\{-[q_\perp d|n-n'| + \mathrm{i}q_z d(n-n')]\} \cdot \tag{31}$$

The sum over n' in the above expression can be evaluated exactly [45] to give

$$S(q_\perp, q_z) = \frac{\sinh(q_\perp d)}{\cosh(q_\perp d) - \cos(q_z d)}. \tag{32}$$

The summation over m and m' in equation (30) can also easily be performed with the help of the Graf theorem for the Bessel functions:

$$J_\nu(R)\exp(i\nu\psi) = \sum_{k=-\infty}^{\infty} J_k(\rho) J_{\nu+k}(r)\exp(\mathrm{i}k\varphi), \tag{33}$$

where

$$R = \sqrt{r^2+\rho^2-2r\rho\cos\varphi}, \exp(\mathrm{i}2\psi) = \frac{r-\rho\exp(-\mathrm{i}\varphi)}{r-\rho\exp(\mathrm{i}\varphi)}. \tag{34}$$

As a result, we finally arrive at the following equation, which determines the collective excitations of the system,

$$1 - \frac{2\pi e^2}{\kappa_0 q_\perp} S(q_\perp, q_z) \sum_{n,n'} J^2_{n-n'}(Z) \Pi_{nn'}(\omega, q_\perp) = 0. \tag{35}$$

Here

$$Z = \frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right| \tag{36}$$

and $\Pi_{nn'}(\omega, q_\perp)$ is the irreducible (proper) polarization function that describes the renormalization of the Coulomb interaction on account of the dynamic screening, having the form

$$\Pi_{nn'}(\omega, q_\perp) = (2\pi a_B^2)^{-1} \sum_{l,l'} P_{ln,l'n'}(\omega) |F_{ll'}(q_\perp a_B)|^2. \tag{37}$$

Note that $\Pi_{nn'}(\omega, q_\perp)$ is purely real, and hence all the collective modes of the system are free of Landau damping. This is due to the fact that the mean value of the electron velocity in any direction vanishes in the case of the purely discrete electron energy spectrum that we consider in this paper.

Thus, in order to obtain the dispersion relation of collective excitations, we need to solve equation (35). It is a difficult equation to solve exactly because it contains infinite sums which, in general, cannot be calculated analytically. Nevertheless, the analytical solution of the problem may be obtained explicitly at low electron densities ($n_s \sim 10^8 - 10^9$ cm^{-2}), so that the carriers are nondegenerate ($\varepsilon_F \ll k_B T$, where ε_F is the Fermi energy) even at low temperatures which are required that the conditions (8) should be satisfied. In this case, we succeed in recasting (see the following) the second term of equation (35) into the more convenient form of a double series, allowing us to extract the collective-mode dispersion relation from equation (35) in a rather simple way. A more sophisticated calculation is needed to obtain analytic closed-form solutions for collective excitations in the case of degenerate electrons ($\varepsilon_F \gg k_B T$), which occurs in n-doped SL systems. This is the subject of ongoing research, which will be presented in a separate paper later.

3 Dispersion relations and properties of collective excitations

In what follows, we restrict our treatment to a nondegenerate electron gas. In this case, the electron distribution function in the presence of an electric field and a magnetic field has the Boltzmann form

$$f^{(0)}(\varepsilon_{ln}) = \exp\left(\frac{\zeta - \varepsilon_{ln}}{k_B T}\right), \tag{38}$$

with $\zeta = \mu + eEz_\lambda$, where μ is the chemical potential proper and $z_\lambda = \langle\lambda|z|\lambda\rangle$ represents the mean z-coordinate of an electron in state $|\lambda\rangle$, which is equal to z_n in our model. Under thermal equilibrium conditions, the magnitude ζ is determined by the requirement that the electron concentration n_s per unit area in each layer of the SL should not change on applying the above-mentioned external fields. It is easy to show that the distribution function (38) normalized on the total number of electrons in the system is then given by

$$f^{(0)}(\varepsilon_l) = 2\pi a_B^2 n_e d\,[1-\exp(-2\alpha)]\exp(-2l\alpha), \tag{39}$$

where we have introduced the parameter $\alpha = \hbar\omega_c/2k_B T$ and the "effective" three-dimensional electron density of the SL n_e defined by $n_e = n_s/d$.

After some algebraic manipulations, which are outlined in Appendix A, we arrive at the final equation, from which the dispersion relations of the collective excitations can be derived,

$$\begin{aligned} 1 - \Omega_p^2(q_\perp, q_z)\frac{2}{u}\exp(-u\coth\alpha) &\sum_{N=1}^{\infty} N\sinh(N\alpha) \\ &\times I_N\left(\frac{u}{\sinh\alpha}\right)\Bigg\{ J_0^2(Z)\frac{1}{\omega^2 - N^2\omega_c^2} \\ &+ 2\sum_{M=1}^{\infty} J_M^2(Z)\sum_{N=1}^{\infty} N\sinh(N\alpha) I_N\left(\frac{u}{\sinh\alpha}\right) \\ &\times \Bigg\{ J_0^2(Z)\frac{1}{\omega^2 - N^2\omega_c^2} + 2\sum_{M=1}^{\infty} J_M^2(Z) \\ &\times \frac{\omega^2 - (N^2\omega_c^2 - M^2\omega_s^2)}{[\omega^2 - (N\omega_c + M\omega_s)^2][\omega^2 - (N\omega_c - M\omega_s)^2]}\Bigg\} = 0. \end{aligned} \tag{40}$$

Here

$$\Omega_p^2(q_\perp, q_z) = \omega_p^2 S(q_\perp, q_z)\frac{q_\perp d}{2} \tag{41}$$

is the plasmon dispersion relation obtained by Das Sarma and Quinn [30] for the SL in the case of zero electric and

magnetic fields. In the above equation ω_p^2 corresponds to the square of the usual plasma frequency for a homogeneous three-dimensional electron gas with an effective electron mass $m_\perp$, that is,

$$\omega_p^2 = \frac{4\pi n_e e^2}{\kappa_0 m_\perp}. \tag{42}$$

Equation (40) can no longer be simplified. In what follows, we solve this equation separately for two qualitatively distinct cases. In the first case ($q_z = 0$) the electron densities in all the layers of the SL oscillate in phase, whereas in the second case ($q_z \neq 0$) the density oscillation phase in the different layers is different. We begin with the first case which is much easier to investigate than the second one.

3.1 Collective-mode dispersion relation for $q_z = 0$

For $q_z = 0$ equation (40) takes the form

$$G(\omega) = 1, \tag{43}$$

where $G(\omega)$ stands for

$$\Omega_p^2(q_\perp, 0)\frac{2}{u}\exp(-u\coth\alpha)\sum_{N=1}^{\infty}\sinh(N\alpha) \times I_N\left(\frac{u}{\sinh\alpha}\right)\frac{N}{\omega^2 - N^2\omega_c^2}. \tag{44}$$

Note that (44) is independent of E. Hence, the $q_z = 0$ modes are insensitive to the electric field strength. The reason for this is quite evident, since the excitations with zero wavevector along the SL axis correspond to density oscillations in the x-y plane. It is not surprising, therefore, that in this case the collective modes are unaffected by the electric field which quantizes the motion of the electrons in the z direction. However, if q_z is slightly different from zero, so that the modes are propagating in a direction slightly off the SL axis, this is no longer true, and the frequencies of the modes do depend on E, in the manner shown further below (Sect. 3.2).

The above expression for $G(\omega)$ shows that the terms in the series in (44) have poles at $\omega = N\omega_c$. It is easy to see that in the vicinity of the singular points the function $G(\omega)$ behaves as follows:

$$\begin{aligned} G(\omega) &\to -\infty \quad \text{for } \omega \to N\omega_c - 0, \\ G(\omega) &\to +\infty \quad \text{for } \omega \to N\omega_c + 0. \end{aligned} \tag{45}$$

If the function $G(\omega)$ is plotted *versus* ω for various values of $q_\perp$, then the roots of the dispersion equation (43) are the intersections of a G *vs.* ω curves with the horizontal line $G(\omega) = 1$. Taking into account (45), one may then state that this equation has an infinite set of real roots

$$\omega = \omega_N(q_\perp), \quad N = 1, 2, \ldots, \tag{46}$$

the Nth root lying in the interval $N\omega_c < \omega_N(q_\perp) < (N+1)\omega_c$.

The explicit form of the collective-mode dispersion relation $\omega_N(q_\perp)$ can be obtained when ω is close to the above-mentioned poles in the series in (44), that is, if the inequality $|\omega - N\omega_c| \ll N\omega_c$ is satisfied. In this case, we retain only the "resonant" term in the series and obtain

$$\omega_N^2(q_\perp) = N^2\omega_c^2[1 + \xi_N(q_\perp)], \tag{47}$$

where

$$\xi_N(q_\perp) = \frac{\Omega_p^2(q_\perp, 0)}{N\omega_c^2}\frac{2}{u}\exp(-u\coth\alpha) \times \sinh(N\alpha) I_N\left(\frac{u}{\sinh\alpha}\right). \tag{48}$$

The general result for the collective-mode dispersion in equation (48) can be further simplified when $q_\perp a_B$, $q_\perp d \ll 1$ (the long-wavelength limit) and when $q_\perp a_B$, $q_\perp d \gg 1$ (the short-wavelength limit). In the long-wavelength limit, the structure factor $S(q_\perp, q_z)$ defined by equation (32) assumes the form $S(q_\perp, 0) \simeq 2/q_\perp d$ for $q_z = 0$. In the same limit, the argument of the Bessel function in equation (48) is much smaller than unity, and hence the power series expansion of $I_N(z)$ can be successfully used here. Retaining only the leading order term in the expansion yields

$$I_N(z) \simeq \frac{1}{N!}\left(\frac{z}{2}\right)^N. \tag{49}$$

Substituting equation (49) into equation (48) and taking into account that $\coth\alpha \simeq 1$ for $\alpha \gg 1$, we obtain

$$\xi_N(q_\perp) \simeq \frac{\omega_p^2}{\omega_c^2}\frac{u^{N-1}}{N^2(N-1)!}. \tag{50}$$

In the short-wavelength limit, the structure factor $S(q_\perp, q_z) \simeq 1$ independently on the values of the parameter q_z. With the use of the following well-known asymptotic expansion of the Bessel function $I_N(z)$ for a large argument (*i.e.*, in the given case, for $u/\sinh\alpha \gg 1$),

$$I_N(z) \simeq \frac{e^z}{\sqrt{2\pi z}}, \tag{51}$$

we have

$$\xi_N(q_\perp) \simeq \frac{\omega_p^2}{N\omega_c^2}\frac{q_\perp d}{4\sqrt{\pi u^3}}\exp(-u)\exp\left[\alpha\left(N + \frac{1}{2}\right)\right]. \tag{52}$$

Going back to the long-wavelength limit of the dispersion relation (47), it is worth noticing that in this limit the obtained solution for $N = 1$ corresponds to the principle magnetoplasmon mode which occurs at the so-called upper-hybrid plasma frequency $\omega_1(q_\perp)$ connected with both the cyclotron frequency ω_c and the plasma frequency ω_p. The expression for $\omega_1(q_\perp)$ given by equation (47), taken jointly with equation (50), is

$$\omega_1(q_\perp) = \left(\omega_p^2 + \omega_c^2\right)^{1/2}[1 + \xi_1(q_\perp)], \tag{53}$$

where

$$\xi_1(q_\perp) = \frac{\omega_p^2}{24(\omega_p^2 + \omega_c^2)}q_\perp^2\left(d^2 + \frac{6\omega_p^2}{\omega_p^2 - 3\omega_c^2}a_B^2\right). \tag{54}$$

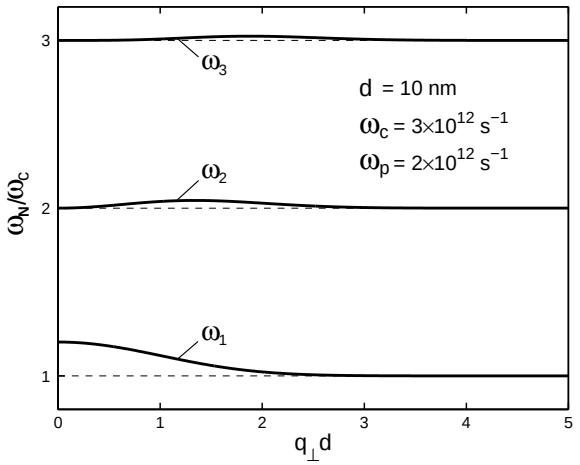

Fig. 1. Plot of ω_N/ω_c given by equation (47) as a function of $q_\perp d$ at the liquid-helium temperature for the case $\omega_p \sim \omega_c$. The SL parameters used to generate this figure are given in the text. Only the principle magnetoplasmon mode branch (ω_1) and the two lowest Bernstein-like mode branches ($\omega_{2,3}$) are shown.

It follows from equation (53) that the frequency $\omega_1(q_\perp)$ is very close to the cyclotron frequency ω_c for the case of low electron density when $\omega_p \ll \omega_c$. On the other hand, the solution (47) for $N = 2, 3, \ldots$ corresponds to the Bernstein-like modes associated with the charge density oscillations occurring at the higher multiples of the cyclotron frequency. In the limiting case of $q_\perp \to \infty$ all the frequencies ω_N approach $N\omega_c$ whatever the electron density might be.

In Figure 1, the dispersion relation results of equation (47) are presented for the case, $\omega_p \sim \omega_c$. Only the principal magnetoplasmon mode (ω_1) and the two lowest Bernstein-like modes ($\omega_{2,3}$) are shown in this plot. In the numerical calculations, we employ the usual parameters for a GaAs-based SL [47]: the electron effective masses $m_\perp = 1.25\, m_\parallel$ and $m_\parallel = 0.079\, m_0$ (m_0 is the free-electron mass), the dielectric constant $\kappa_0 = 12.5$, and the SL period $d = 10$ nm. The plot of the three modes in Figure 1 shows that the lowest branch exhibits an appreciable variation with $q_\perp d$ in the region $q_\perp d \leq 1$, whereas the higher branches reveal but very weak dispersion. On the other hand, when $\omega_p \ll \omega_c$ we observe (the plot is not shown) no dispersion for all the modes pictured in Figure 1, as expected from the above discussion.

3.2 Collective-mode dispersion relations for $q_z \neq 0$

We now consider the spectrum of collective modes with non-zero wavevector along the SL growth axis, *i.e.* with $q_z \neq 0$. In this case a SL system supports two kinds of collective excitations which are described by the dispersion equation (40). The first of them corresponds to the electron density oscillating at or near the cyclotron frequency and its higher multiples. Actually, they are the usual electronic cyclotron (or Bernstein) modes except that now their dispersion is affected by the electric field. Another type of excitations involved in equation (40) is a new kind of collective modes, which we call cyclotron-Stark modes in this paper, and which arise only upon application of electric and magnetic fields parallel to the SL axis. These extra collective modes are related to the electron transitions between different Wannier-Stark-Landau ladder levels in such a system. The possibility of their existence is evident from equation (40) where the terms in the double series have the "resonant" energy denominator of the type $(\omega - \omega_{NM}^{\pm})^{-1}$ with $\omega_{NM}^{\pm} = N\omega_c \pm M\omega_s$.

We first examine the cyclotron modes with frequencies close to $N\omega_c$. There are two cases to consider, depending on the ratio between the two frequencies ω_c/ω_s: rational (ω_c and ω_s are commensurate) and irrational (ω_c and ω_s are incommensurate). We look at the second case first. Since we are concerned only with the frequency range near $N\omega_c$, the second term in the curly brackets of equation (40) can be neglected. The reason for this is that the double series in equation (40) contains no "resonant" terms of the type $(\omega - N\omega_c)^{-1}$ for any values of N and M. Under a certain condition (see below), the neglect of the above-mentioned term is justified for the rational ω_c/ω_s as well, in spite of the fact that in this case among the indices N and M in equation (40) such N_0 and M_0 that $N_0\omega_c \pm M_0\omega_s = N\omega_c$ for certain N should necessarily be found. Then, the double series in equation (40) will contain the particular term proportional to $(\omega - N\omega_c)^{-1}$, which will give the major contribution to the sum if $|\omega - N\omega_c| \ll N\omega_c$. Nevertheless, provided the inequality $|q_z|d \ll \pi$ is satisfied, the terms in the double series of equation (40) which are proportional to the Bessel function $J_M(Z)$ with $M \geq 1$ can be neglected, since they lead to but a small correction to the dispersion of the modes obtained further below. Indeed, under actual experimental circumstances, the parameter $\Delta/\hbar\omega_s$ is not too large ($\sim 3 - 5$), so that only a comparatively small number of Wannier-Stark levels can be arranged on the width of the ground miniband. In this case the argument of the Bessel function $J_M(Z)$ will be small compared to unity, if we confine our consideration to the region of small q_z (in the sense that $|q_z| \ll \pi/d$). Since the Bessel function $J_M(Z)$ falls off rapidly for $Z \ll 1$, the second term in the curly brackets of equation (40) is small compared to the first one, so that we have to retain only the latter. In keeping only the "resonant" term therein, we finally arrive at the following dispersion relation:

$$\omega_N^2(q_\perp, q_z) = N^2\omega_c^2[1 + D_N(q_\perp, q_z)], \quad N = 1, 2, \ldots, \tag{55}$$

where the dispersion coefficient $D_N(q_\perp, q_z)$ is given by

$$D_N(q_\perp, q_z) = \frac{\Omega_p^2(q_\perp, q_z)}{N\omega_c^2}\frac{2}{u}\exp(-u\coth\alpha)\sinh(N\alpha) \times I_N\left(\frac{u}{\sinh\alpha}\right) J_0^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right). \tag{56}$$

It is clear from the discussion above that equation (55) is valid for all q_z if the frequencies ω_c and ω_s are incommensurate and for small q_z ($|q_z| \ll \pi/d$) if they are commensurate.

The asymptotic behavior of the dispersion coefficient in equation (55) can readily be obtained by appropriate expansion of the Bessel function $I_N(z)$. Using equations (49) and (51), we get after simple algebra

$$D_N(q_\perp, q_z) = \frac{u^{N-1}}{2N^2(N-1)!}\frac{\omega_p^2}{\omega_c^2}\frac{(q_\perp d)^2}{1-\cos q_z d} \times J_0^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right). \quad (57)$$

in the long-wavelength limit ($q_\perp a_B, q_\perp d \ll 1$) and

$$D_N(q_\perp, q_z) = \frac{\omega_p^2}{N\omega_c^2}\frac{q_\perp d}{4\sqrt{\pi u^3}}\exp(-u)\exp\left[\alpha\left(N+\frac{1}{2}\right)\right] \times J_0^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right). \quad (58)$$

in the short-wavelength limit ($q_\perp a_B, q_\perp d \gg 1$).

It follows from equation (55) that in varying q_z within the first Brillouin minizone of the SL ($-\pi/d \le q_z \le \pi/d$), the collective excitations with $q_\perp$ fixed will evolve in frequency to form continuous spectral bands which are symmetric with respect to q_z in virtue of the equality $\omega_N(q_\perp, -q_z) = \omega_N(q_\perp, q_z)$. The broadening of the frequencies $\omega_N(q_\perp, q_z)$ into continua arises from the q_z-dependence of $\Omega_p^2(q_\perp, q_z)$ and $J_0^2(Z)$ in equation (56). It is easy to see that, since $\tanh(q_z d/2) \le S(q_\perp, q_z) \le \coth(q_z d/2)$ and $J_0^2(Z) \le 1$, the lower bound of the Nth band labeled ω_N^- and the upper bound of the same band labeled ω_N^+ correspond to out-of-phase ($q_z = \pm\pi/d$) and in-phase ($q_z = 0$) charge density oscillations in adjacent layers of the SL, respectively. This is illustrated in Figure 2 where the dimensionless frequencies ω_N/ω_c of the first two cyclotron modes ($N = 1, 2$) are plotted against $q_\perp d$ for the case corresponding to that presented above in Figure 1. The parameters used in the calculation are the same as in Figure 1, except that now q_z is varying from zero out to the effective Brillouin zone edge π/d. Our calculation shows that for small $q_\perp d$, the width of the lower band is at a maximum, whereas for large $q_\perp d$, it becomes extremely small. For the higher bands originating from Bernstein-like modes, the maximum of the bandwidth occurs for a larger value of $q_\perp d$. However, it should be noted that on the whole the bandwidths are very small, particularly for electron densities corresponding to the most realistic experimental situation where the plasmon frequency is much smaller than the cyclotron one. In consequence, as seen from Figure 2, there is a gap between the first and the second bands for all the $q_\perp d$ values considered. It can be shown that the gaps in the spectrum of collective modes occur between any adjacent bands arising from higher cyclotron mode branches with $N = 3, 4, \ldots$

The general result (55) for the dispersion relation shows that the frequency of the cyclotron modes with nonzero q_z exhibit oscillatory variations as a function of electric field strength, which are due to oscillatory behavior of the Bessel function in equation (56). The physical origin of these oscillations is evident. They can be attributed to the geometrical resonance between the wavelength λ_0 of the collective mode with $q_z = 2\pi/\lambda_0$ and the amplitude $A_0 = \Delta/2eE$ of a Bloch oscillation of an electron undergoing Bragg diffraction on a periodic potential in the presence of an intense electric field. In Figure 3 we display the dispersion of ω_N for $N = 4$ as a function of electric field E, obtained with equations (55, 56) for a fixed value of q_z which we have chosen so that the condition of the validity of equation (55) should still be satisfied. The first oscillation of the considered cyclotron mode

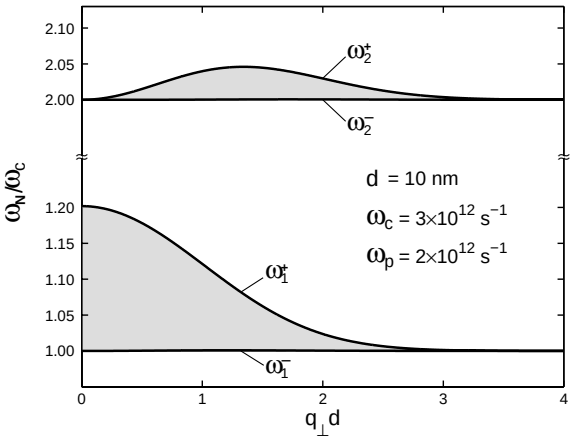

Fig. 2. Plot of ω_N/ω_c given by equation (55) as a function of $q_\perp d$ for the same case as in Figure 1. The parameters are the same as those used to generate Figure 1, except that q_z varies here from 0 to π/d. The figure shows the two lowest collective-mode bands (the shaded areas) corresponding to the principle magnetoplasmon mode ($N = 1$) and the first Bernstein-like mode ($N = 2$).

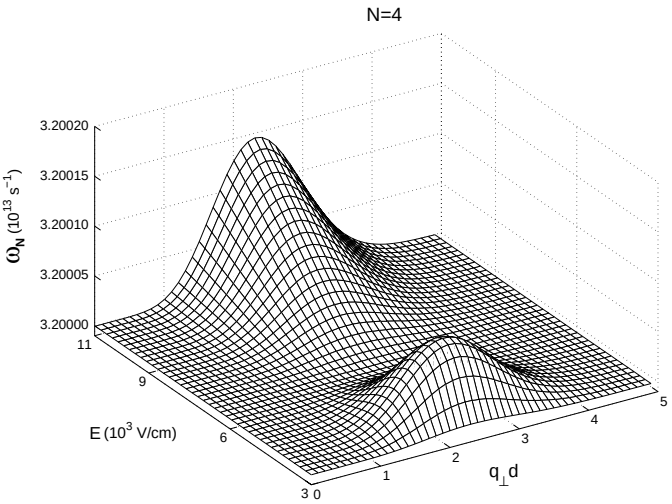

Fig. 3. A three-dimensional plot of the cyclotron mode ω_N ($N = 4$) given by equation (55) as a function of E and $q_\perp d$. The parameters used in the calculation are $n_s = 4.5 \times 10^8$ cm^{-2}, $T = 4.2$ K, $B = 5$ T and $q_z = 0.7 \times 10^6$ cm^{-1}. Others are the same as in Figure 1. The figure demonstrates the oscillatory behavior of the frequencies of the cyclotron modes as a function of electric field strength as well as their non-monotonic dependence on $q_\perp d$ at a fixed value of E.

frequency ω_4 is clearly seen in the region where $q_\perp d \sim 1$. Our calculations also show that the oscillations are shifted towards larger values of $q_\perp d$ as N increases. On the other hand, the oscillations fail to manifest themselves both for $q_\perp d \ll 1$ and $q_\perp d \gg 1$, since the amplitude of the oscillating term in equation (55) falls off rapidly in these two regions. However, it should be noted that even in the region $q_\perp d \sim 1$ the relative change of the cyclotron mode frequency is very small ($\sim 10^8$ s^{-1}). Hence, it will at least be extremely hard, if not impossible to measure these oscillations, particularly if one takes into account that in real systems there always exist collision effects which will lead to damping of the modes at finite q.

We now turn our attention to the study of another type of collective modes which we call cyclotron-Stark modes. We assume that $N\omega_c \neq M\omega_s$, since the case $N\omega_c = M\omega_s$ is apparently reduced to the one that we considered above. To obtain an analytical expression for the frequencies of the modes, we use equation (40) in the near vicinity of $\omega^{\pm}_{NM}$, or more specifically $|\omega - \omega^{\pm}_{NM}| \ll \omega^{\pm}_{NM}$. In this case, we can neglect the first term in the curly brackets of equation (40), whereas in the second term we have to keep only the "resonant" one. We then get

$$[\omega^{\pm}_{NM}(q_\perp, q_z)]^2 = (\omega^{\pm}_{NM})^2[1 + D^{\pm}_{NM}(q_\perp, q_z)] \qquad (59)$$

$(N, M = 1, 2, \dots)$, where the dispersion coefficient $D^{\pm}_{NM}(q_\perp, q_z)$ is given as

$$D^{\pm}_{NM}(q_\perp, q_z) = \frac{\Omega_p^2(q_\perp, q_z)}{\omega^{\pm}_{NM}\omega_c}\frac{2}{u}\exp(-u\,\coth\alpha)\sinh(N\alpha) \times I_N\left(\frac{u}{\sinh\alpha}\right) J_M^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right). \qquad (60)$$

The dispersion relation (59) is the main result of this paper. All collective excitations of the system are contained in equation (59) if we considered it to be valid for cases $M = 0$ and $q_z = 0$. Indeed, for the $M = 0$ case the above equation is formally identical to equation (55) which describes the spectrum of the cyclotron modes propagating in "oblique" direction ($q_z \neq 0$) with respect to the SL axis, whereas for the $M = 0$, $q_z = 0$ case equation (59) reduces to equation (47) which defines the spectrum of the collective modes propagating perpendicular to the SL axis. Thus, equation (65) can serve as the basic dispersion relation for the collective excitations of the system under consideration. We therefore write down the asymptotic expansions for $D^{\pm}_{NM}(q_\perp, q_z)$ which are valid in the following two limiting cases. For small $q_\perp$ (such that $q_\perp a_B$, $q_\perp d \ll 1$), equation (60) gives

$$D^{\pm}_{NM}(q_\perp, q_z) = \frac{u^{N-1}}{2N!}\left\{\delta(q_z,0)\delta(M,0)\frac{2\omega_p^2}{N\omega_c^2} + [1-\delta(q_z,0)]\frac{\omega_p^2}{\omega^{\pm}_{NM}\omega_c}\frac{(q_\perp d)^2}{1-\cos q_z d}\,J_M^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right)\right\}. \qquad (61)$$

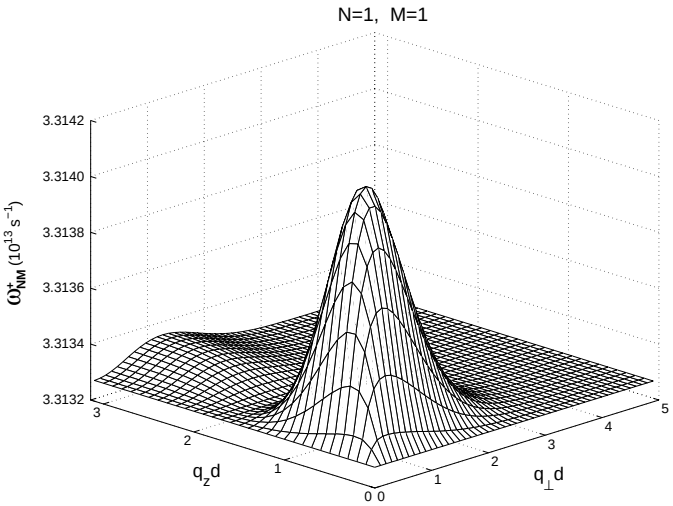

Fig. 4. A three-dimensional plot of the lowest ($N = 1$, $M = 1$) cyclotron-Stark mode ω^{+}_{NM} given by equation (59) as a function of $q_\perp d$ and $q_z d$ for the ratio between the frequencies $\omega_s/\omega_c = \pi$. Other parameters are the same as in Figure 3.

In the opposite limit where $q_\perp a_B$, $q_\perp d \gg 1$, we have

$$D^{\pm}_{NM}(q_\perp, q_z) = \frac{q_\perp d}{4\sqrt{\pi u^3}}\exp(-u)\exp\left[\alpha\left(N+\frac{1}{2}\right)\right] \times\left\{\delta(q_z,0)\delta(M,0)\frac{\omega_p^2}{N\omega_c^2} + [1-\delta(q_z,0)]\frac{\omega_p^2}{\omega^{\pm}_{NM}\omega_c}\,J_M^2\left(\frac{\Delta}{\hbar\omega_s}\left|\sin\frac{q_z d}{2}\right|\right)\right\}. \qquad (62)$$

Here, as before, $\delta(a, b)$ denotes Kronecker's delta.

Figure 4 shows the dispersion of the lowest ($N = 1$, $M = 1$) collective mode ω^{+}_{NM} calculated from equation (59) for the same SL parameters as in Figure 1. Since we examine an irrational case when the frequencies ω_c and ω_s are incommensurate, we have chosen (rather arbitrary) the ratio between the frequencies ω_c/ω_s to be equal to π. The dispersion relations of the modes labeled $\omega^{-}_{NM}(q_\perp, q_z)$ in equation (59) are graphed in Figure 5 for the same values of N and M as in Figure 4 and also for the case $N = 4, M = 1$. The examples presented in the above figures are typical of dispersion of the cyclotron-Stark modes predicted in this paper, and therefore they illustrate the specific features of dispersion of higher collective modes of this type as well.

It follows from equation (59) that at fixed values of $q_\perp$ and q_z the frequencies of the cyclotron-Stark modes have an oscillatory dependence on the electric field strength. The origin of this effect is the same as discussed above for the cyclotron modes $\omega_N(q_\perp, q_z)$. In addition, equation (59) shows that the frequencies $\omega^{\pm}_{NM}(q_\perp, q_z)$, with $q_\perp$ fixed, oscillate with changing q_z. In Figures 4 and 5 at least one oscillation of this kind can be observed in the region $q_\perp d \sim 1$. These oscillations are due to the oscillatory variation of the Bessel function $J_M(Z)$ in equation (60) as q_z changes. Inspection of Figures 4 and 5 shows that the principle extreme points of both functions $\omega^{+}_{NM}(q_\perp, q_z)$

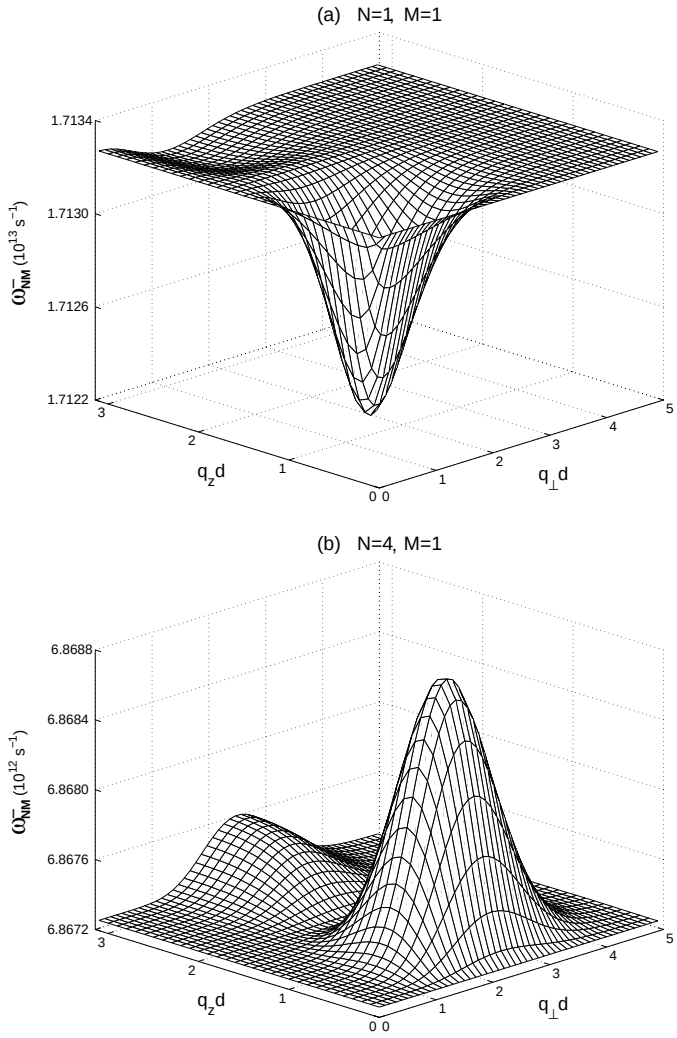

Fig. 5. A three-dimensional plot of ω^-_{NM} given by equation (59) as a function of $q_\perp d$ and $q_z d$ for the same values of (N, M) as in Figure 4 (panel (a)) and for $N = 4$, $M = 1$ (panel (b)). Parameters are the same as in Figure 4. The figure demonstrates the different nature of the dispersion of the Stark-cyclotron mode branches ω^-_{NM}, with q_z fixed, in the regions $q_\perp d \ll 1$ and $q_\perp d \gg 1$, depending on the values of N and M.

and $\omega^-_{NM}(q_\perp, q_z)$ are situated at $q_z d \simeq 1$, $q_\perp d \sim 1$. As a general trend we find from the numerical calculations that with an increase in q_z the extreme points are shifted towards larger values of the parameter $q_\perp d$. In this region the amplitude of the oscillations of $\omega^\pm_{NM}(q_\perp, q_z)$ becomes extremely small. That is why the second and the following oscillations of the frequencies $\omega^\pm_{NM}(q_\perp, q_z)$ are not seen in Figures 4 and 5.

One should notice an interesting feature of the obtained spectra of the cyclotron-Stark modes. In Figures 4 and 5, we can clearly observe the non-monotonic dependence of the frequencies $\omega^\pm_{NM}(q_\perp, q_z)$ on the parameter $q_\perp d$ in the region $q_z d \simeq 1$. Such behavior of $\omega^\pm_{NM}(q_\perp, q_z)$ is due to the fact that the different factors depending on $q_\perp$ in equation (60) change differently as $q_\perp d$ increases. A general feature of the curves in Figure 4 is that as $q_\perp d$ varies from 0 to 5 the frequencies $\omega^+_{NM}(q_\perp, q_z)$, with q_z fixed, first increase starting from $\omega^+_{NM}(0, q_z)$, reach a maximum, and then decrease approaching $\omega^+_{NM}(0, q_z)$ in the short-wavelength limit. In other words, all the mode branches $\omega^+_{NM}(q_\perp, q_z)$ are characterized by normal dispersion for small $q_\perp$ ($q_\perp d \ll 1$) and by anomalous dispersion in the opposite limit of large $q_\perp$ ($q_\perp d \gg 1$). In contrast, the dispersion of the branches $\omega^-_{NM}(q_\perp, q_z)$ in the above limiting cases can be normal as in Figure 5b or anomalous as in Figures 5a depending on the values of N and M. This is due to the fact that the sign of the dispersion coefficient $D^-_{NM}(q_\perp, q_z)$ can be either positive or negative depending on the sign of the frequency $\omega^-_{NM} = N\omega_c - M\omega_s$ in the denominator of equation (60). For the ratio of the frequencies $\omega_c/\omega_s = \pi$, which have been chosen for the numerical calculations, the positive sign occurs for the $N = 4, M = 1$ case, whereas the negative sign occurs for the $N = 1$, $M = 1$ case. This explains the change in the nature of dispersion for the cyclotron-Stark mode branches $\omega^-_{NM}(q_\perp, q_z)$ in Figure 5b as compared to those in Figures 5a.

4 Summary and concluding remarks

In this paper, we have studied in detail the collective electronic excitations in a semiconductor SL with electric and magnetic fields parallel to the SL axis. Such an orientation of the fields is particularly interesting because in this case the electron energy spectrum is purely discrete, consisting of the Landau and Wannier-Stark ladder levels.

We have used the equation-of-motion method to derive the dispersion relations of the collective excitations at relatively low electron concentration where the electron plasma can be treated as a nondegenerate electron gas in the RPA. This is of relevance for the electron gas in undoped SLs of the GaAs/Al_xGa_{1-x}As type.

We predict that the spectrum of the collective modes propagating perpendicular to the SL axis consists of a principle magnetoplasmon mode (at a frequency very close to the cyclotron frequency ω_c) and an infinite set of Bernstein-like modes associated with the higher multiples of ω_c. All these modes are practically dispersionless and are not affected by Wannier-Stark quantization.

The spectrum of the collective excitations propagating in "oblique" direction with respect to the SL axis is more complicated. In this case, in addition to the cyclotron modes with frequencies close to $N\omega_c$, we predict a completely new kind of collective modes at hybrid frequencies $|N\omega_c \pm M\omega_s|$, with N, M arbitrary integers, which we call cyclotron-Stark modes. In the case of non-zero wavevector component along the SL axis, the frequencies of these modes exhibit an oscillatory dependence on the electric field strength. We infer from our calculations that the amplitude of the oscillations is on the order of 10^8 s^{-1}. This suggests that the oscillations are probably not so simple to observe, since in SLs the collision linewidths are usually much bigger.

$$\Pi^{\pm}_{nn'}(\omega,q_\perp) = \frac{n_e d}{\hbar}[1-\exp(-2\alpha)]\exp(-u)\left\{\sum_{N=0}^{\infty}\frac{u^N \mathrm{e}^{-2\alpha N}}{\pm(n'-n)\omega_s \mp \omega + N\omega_c}\sum_{l'=0}^{\infty}\frac{l'!}{(l'+N)!}\exp(-2l'\alpha)[L^N_{l'}(u)]^2 + \sum_{N=1}^{\infty}\frac{u^N}{\pm(n'-n)\omega_s \mp \omega - N\omega_c}\sum_{l=0}^{\infty}\frac{l!}{(l+N)!}\exp(-2l\alpha)[L^N_l(u)]^2\right\}, \quad (A.4)$$

Another interesting feature of the collective modes considered in this paper is that they are free of Landau damping. As mentioned above, this is caused by the full discreteness of the electron energy spectrum. Without Landau damping the collective modes decay due to the electron scattering processes on phonons and different lattice imperfections. If such collision effects are included, then there appears a small imaginary part of the polarization function $\Pi_{nn'}(\omega,q_\perp)$, which determines the response of the system to an external AC electric field. Therefore the specific predictions made in this paper for the collective excitations should be observable in Raman light scattering experiments. To enhance the sensitivity of the method, such a scattering is usually observed in the resonant regime where the frequency of the incident light beam (ω_L) is close to the fundamental absorption edge of SLs [26–48]. In the case under consideration the scattered-radiation spectrum should exhibits peaks at the frequencies $\omega_L \pm \omega^{\pm}_{NM}$, $\omega_L \pm 2\omega^{\pm}_{NM}$ and the like, the width of the peaks being determined by the collective mode damping $1/\tau$. It should be noted that in the case when cyclotron ω_c and Wannier-Stark ω_s frequencies are incommensurate, the hybrid frequency $\omega^{\pm}_{NM}$ is defined by two "quantum" numbers N and M simultaneously, and hence at large values of N and M the sequence of lines corresponding to the cyclotron-Stark collective modes in the scattered-radiation spectrum can seem to be chaotic. The experimental observation of such a kind of chaos in Raman scattering spectra in SLs should present a significant interest from the standpoint of general physics.

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Appendix A

Substituting equation (24) into equation (37), the expression for $\Pi_{nn'}(\omega,q_\perp)$ can be written in the following convenient form:

$$\Pi_{nn'}(\omega,q_\perp) = \Pi^{+}_{nn'}(\omega,q_\perp) + \Pi^{-}_{nn'}(\omega,q_\perp), \quad (A.1)$$

where $\Pi^{\pm}_{nn'}(\omega,q_\perp)$ is given by

$$\Pi^{\pm}_{nn'}(\omega,q_\perp) = \frac{1}{2\pi a_B^2\hbar}\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}|F_{ll'}(q_\perp a_B)|^2 f^{(0)}(\varepsilon_l) \times [(l-l')\omega_c \pm (n'-n)\omega_s \mp \omega]^{-1}. \quad (A.2)$$

To evaluate $\Pi_{nn'}(\omega,q_\perp)$, we recast the sums of the type given in equation (A.2) as follows:

$$\sum_{l=0}^{\infty}\sum_{l'=0}^{\infty}A_{ll'} = \sum_{l=0}^{\infty}\left(\sum_{l'=0}^{l}A_{ll'} + \sum_{l'=l+1}^{\infty}A_{ll'}\right) = \sum_{l=l'}^{\infty}\sum_{l'=0}^{\infty}A_{ll'} + \sum_{l'=l+1}^{\infty}\sum_{l=0}^{\infty}A_{ll'}. \quad (A.3)$$

We now transform the last line of equation (A.3) by changing the summation indices; for the first term where $l \geq l'$ we change $l \to l-l'$, while for the second term where $l < l'$ we change $l' \to l'-l$. With the aid of equations (26) and (39) of the main text, we then get

see equation (A.4) above,

where $u = q_\perp^2 a_B^2/2$ and $N = |l-l'|$.

The series summations over l and l' in equation (A.4) can easily be performed by using the bilinear generating function for the associated Laguerre polynomials

$$(1-z)^{-1}\exp\left(-2u\frac{z}{1-z}\right)u^{-p}z^{-p/2}I_p\left(2u\frac{z}{1-z}\right) = \sum_{l=0}^{\infty}\frac{l!}{\Gamma(l+p+1)}z^l[L^p_l(u)]^2, \quad (A.5)$$

where $\Gamma(x)$ is the gamma-function and $I_p(z)$ is a modified Bessel function of the first kind and of order p. The above equation represents the well-known Hille-Hardy formula [46] which is valid for $p > -1$ and all $|z| < 1$. Setting $z = \exp(-2\alpha)$ in equation (A.5) and taking into account that $I_{-p}(z) = I_p(z)$ for integer p, we easily get from equation (A.4) that

$$\Pi_{nn'}(\omega,q_\perp) = \frac{2n_e d}{\hbar}\exp(-u\coth\alpha) \times \sum_{N=-\infty}^{\infty}\frac{\sinh(N\alpha)}{\omega - N\omega_c - (n'-n)\omega_s}I_N\left(\frac{u}{\sinh\alpha}\right). \quad (A.6)$$

Further, we substitute equation (A.6) into equation (35) and change the variable of the summation n' to $M = n'-n$. After performing the summation over n in (35) we get equation (40) of the main text.

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